Statistical Analysis of the Relative Strength of Chess Positions

Recent Advances in Statistical Chess Theory

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SUMMARY

The statistical value of a piece on its occupied square of origin is extended to those squares controlled by the piece. A cumulative strength metric applies to each square controlled by a piece. The computer model, Relative Strength (RS), predicts the next move in an opening example from the world computer chess championship. A predictive rate is significant. RS is a teaching aid for recognition of tactical chess patterns based on positional patterns.

INTRODUCTION

Hans Berliner, a widely recognized chess champion, rated the values of the pieces, of unpassed white pawns at the beginning of the game, of unpassed white pawns at the ending of the game, and of white pawns as they advanced based on structures (The System. London: Gambit Publications Ltd. 1999). Berliner did not publish data from which these statistics are derived. Hence the results of Berliner are irreproducible, unverifiable, and unused here.

A cumulative strength metric applies to each square controlled by a piece. The computer model, Relative Strength (RS), predicts the next move in an opening example from the world computer chess championship. A predictive rate is significant. RS is a teaching aid for recognition of tactical chess patterns based on positional patterns.

PROBLEM STATEMENT

The assignment of arbitrary values to the chess pieces always follows the relative mobility of the pieces. Any system of assignment places more value on the queen than on a rook or a bishop and more value on a bishop or knight than on a pawn. However such systems share a common factor if the basis for assignment is not described and reproducible and is opinion.

APPROACH AND TECHNIQUES

An exact system of assigning values is based on the quantitative mobility of pieces such as counting the number of squares controlled by a piece at all possible destination locations. This basis is reproducible and verifiable. In this paper, the author adds these refinements. The target king is included as a rated fighting piece. Before or after the castling move, the potential destination squares controlled by the king and rooks are not counted twice. For pawns on rank 8, the squares controlled respectively by becoming another piece are not counted before promotion because the pawn is not yet one of those pieces. For a pawn on the next adjacent square in front of its queen or bishop, a diagonal square controlled by the pawn accumulates those piece values behind it.

In Tab. 1 the queen (Q) on any square controls minimally 21 squares for a total of 1456 squares. The rook (R) on any square controls 14 squares and a total of 896 squares. In Tab. 2 the bishop (B) on any square controls minimally seven squares and a total of 560 squares. In Tab. 3 the king (K) on any square controls minimally three squares and a total of 420 squares. In Tab. 4 the knight (N) on any square controls minimally two squares and a total of 384 squares. In Tab. 5 the pawn (P) on any square except the first or second rank controls minimally one square and a total of 84 squares.

Table 1. The queen (Q) on any square

Q 8	21	21	21	21	21	21	21	21
Q 7	21	23	23	23	23	23	23	21
Q 6	21	23	25	25	25	25	23	21
Q 5	21	23	25	27	27	25	23	21
Q 4	21	23	25	27	27	25	23	21
Q 3	21	23	25	25	25	25	23	21
Q 2	21	23	23	23	23	23	23	21
Q 1	21	21	21	21	21	21	21	21
	a	b	c	d	e	f	g	h

Table 2. The bishop (B) on any square

B 8	7	7	7	7	7	7	7	7
B 7	7	9	9	9	9	9	9	7
B 6	7	9	11	11	11	11	9	7
B 5	7	9	11	13	13	11	9	7
B 4	7	9	11	13	13	11	9	7
B 3	7	9	11	11	11	11	9	7
B 2	7	9	9	9	9	9	9	7
B 1	7	7	7	7	7	7	7	7
	a	b	c	d	e	f	g	h

Table 3. The king (K) on any square

K 8	3	5	5	5	5	5	5	3
K 7	5	8	8	8	8	8	8	5
K 6	5	8	8	8	8	8	8	5
K 5	5	8	8	8	8	8	8	5
K 4	5	8	8	8	8	8	8	5
K 3	5	8	8	8	8	8	8	5
K 2	5	8	8	8	8	8	8	5
K 1	3	5	5	5	5	5	5	3
	a	b	c	d	e	f	g	h

Table 4. The knight (N) on any square

N 8	2	3	4	4	4	4	3	2
N 7	3	4	6	6	6	6	4	3
N 6	4	6	8	8	8	8	6	4
N 5	4	6	8	8	8	8	6	4
N 4	4	6	8	8	8	8	6	4
N 3	4	6	8	8	8	8	6	4
N 2	3	4	6	6	6	6	4	3
N 1	2	3	4	4	4	4	3	2
	a	b	c	d	e	f	g	h

Table 5. The pawn (P) on any square

P 8	0	0	0	0	0	0	0	0
P 7	1	2	2	2	2	2	2	1
P 6	1	2	2	2	2	2	2	1
P 5	1	2	2	2	2	2	2	1
P 4	1	2	2	2	2	2	2	1
P 3	1	2	2	2	2	2	2	1
P 2	1	2	2	2	2	2	2	1
P 1	0	0	0	0	0	0	0	0
	a	b	c	d	e	f	g	h

Table 6. Pieces and total values of squares

3	♟		3	♟ **-1**		3	♟ **0**	**14**
2			2			2	**0**	**-1+14 = 13**
1		♖	1		♖ **+ 14**	1	**14**	♖ **0**
	a	b		a	b		a	b

A method for applying the data in Tabs. 1-5 to chessboard positions with pieces requires two steps. The first step assigns each piece a value for its respective location on the chessboard. The second step implies the value of the controlled squares based on the value of the square of origin of the pieces.

For example, consider the position in Tab. 6 for a white rook on b1 in front of a black pawn on a3. For arithmetic purposes, white pieces are set arbitrarily as positive numbers and black pieces as negative numbers. The table values are the white rook at b2 as 14 and the black pawn at a3 as –1. The implied values of the squares are as follows: a3 and b3 become 14 from the rook (14) at b1; and b2 becomes 13 from the pawn (-1) at a3 and the rook (14) at b1. The other squares become 0.

For computerization an arbitrary convention is that squares controlled by white pieces are calculated as positive numbers, and squares controlled by black pieces as negative numbers by a product of –1 of the respective white pieces.

Table 8. Values for pieces after 1. d4 Nf6

8	bR-14	bN-3	bB-7	bQ-21	bK-5	bB-7		bR-14
7	bP-1	bP-2	bP-2	bP-2	bP-2	bP-2	bP-2	bP-1
6						bN-3		
5								
4				wP+2				
3								
2	wP+1	wP+2	wP+2		wP+2	wP+2	wP+2	wP+1
1	wR+14	wN+3	wB+7	wQ+21	wK+5	wB+7	wN+3	wR+14
	a	b	c	d	e	f	g	h

Table 9. Cumulative values from Tab. 8

8	bR	bN-14	bB-21	bQ-5	bK-29	bB-19	-22	bR
7	bP-14	bP-7	bP-21	bP-44	bP-33	bP-5	bP-7	bP-22
6	-12	-3	-7	-11	-11	bN-4	-3	7-9=-2
5			+2	-8	+2		+7	-8
4				wP+21	-8	+7	-8	
3	+12	+3	+5	+32	+9	+7	+3	+12
2	wP+14	wP+7	wP+21	+36	wP+36	wP+5	wP+7	wP+14
1	wR	wN+14	wB+21	wQ+5	wK+21	wB+5	wN+14	wR
	a	b	c	d	e	f	g	h

If another piece of the same color type is blocking any square above, then that square of the other piece acquires the value of the queen (plus the values of any supporting pieces), but the successive squares beyond that square have no value from the queen. If an opposing piece also controls that square of the other piece, then the value of the square becomes the sum of the positive value of the other piece plus the negative value of the opposing piece.

To apply the piece arrays to a position requires repeating steps that modify the array to exclude squares blocked by all other pieces. To determine the cumulative strength value by piece by square of a position, the values of each piece by the squares controlled are summed into a final array. An example follows for the position arising from 1. d4 Nf6 with the arrays for white and black in Tab. 8 and the cumulative values in Tab. 9.

The computer implementation of the above is named RS for relative strength. In RS the values in Tab. 9 are represented graphically in a color range. Blue is assigned to the black side, and red is assigned to the white side. The values are scaled by color based on the square in the position with the largest absolute. For example in Tab. 9, the largest absolute value for any square is at d7 for –44 on the black side. Therefore the color scale extrema range is arbitrarily set at blue for –44, green for 0, and red for +44. The strongest black side squares are blue, the strongest white side squares are red, and for either side the neutral squares are green. For the black side, the colors range from strongest to neutral are blue to blue-green to green. For the white side, the color ranges from strongest to neutral are red to orange-brown to green.

A method to supply positional information about chess pieces on a chessboard is the ASCII text notation. This describes only the location of pieces on squares and nothing else. Sample text of a position is: wKd6, Rc8, g8, Pg2, [or "/"] bKh7, Qe1, Pd4, e3, h5, h6. This is parsed to include white king on d6 and black pawn on h6. The computer program performs this parsing function automatically. The text position may be produced using another computer program named ChessBase 8 where the position is entered graphically onto a new board. The email option is chosen with the file format as text. From the email body the text notation is copied and pasted into an input file for the RS program. These steps avoid the processing of more complex position formats.

To evaluate each position, a statistic is derived from the N-by-M contingency test, of which the χ^2 test is a subset. An advantage of the N-by-M test is that the expected values are derived automatically from the observed values. Raw values of squares for the black side contain negative numbers that may not be processed by the statistical test. Therefore negative values must be scaled to make them positive values. If a scaled value is less than 5 then the value may be too small to be meaningful. If a scaled value is in the range of 5 to 9, then the value may be modified to increase it slightly using the Yates correction.

If a scaled value is 10 or greater than the value is probably large enough for a meaningful test. The largest absolute value for a square surrounded by only black pieces is Q -27, R -14, R -14, B -13, N -8, N -8, K -8, and P -2 for a total of s = -94 and absolute value of s = 94. All squares of the black side should have a scaled value of at least 5. Therefore all values of the squares are arbitrarily scaled by s + 5 = 99 before the statistical test is performed.

RESULTS

Tab. 10 shows piece values based on square mobility total 3800. Tab. 11 shows piece values scaled to one pawn as 1.00. Tab. 12 shows piece values scaled to 100 for tests.

Table 10. Pieces based on square mobility

♕	♖	♗	♔	♘	♙
1456	896	560	420	384	84

Table 11. Pieces scaled to one pawn

♕	♖	♗	♔	♘	♙
13.0	8.0	5.0	3.8	3.4	1.0

Table 12. Pieces scaled to 100

♕	♖	♗	♔	♘	♙
11.3	6.9	4.3	3.3	3.0	0.97

Additional results are a tabulation of most of the first three moves for white and black is established with contingency values. That tabulation is sorted by relative strength and by alphabetical move. Some openings of interest are expanded to include the two best moves of any variation through move six for black. The first two moves considered are sorted alphabetically in Tab. 13. The first two moves considered are sorted by relative strength in Tab. 14. Contingency values are rounded and grouped by units of two into the columnar headings.

Table 13. Moves 1 and 2 sorted by letter

b3	b4	c4	d3	d4	e3	e4	f4	g3	Nc3	Nf3
b3 d5	b4 d6	c4 c5	d3 b6	d4 b6	e3 b6	e4 c5	f4 d6	g3 d5	Nc3 d5	Nf3 d5
b3 d6		c4 c6	d3 d5	d4 c5	e3 g6	e4 c6		g3 d6	Nc3 e5	Nf3 d6
		c4 d5	d3 e5	d4 c6	e3 Nc6	e4 d5			Nc3 c6	Nf3 Nc6
		c4 d6	d3 g6	d4 d5	e3 Nf6	e4 d6			Nc3 Nf6	Nf3 Nf6
		c4 e5	d3 Nf6	d4 d6		e4 e5				
		c4 e6		d4 e5		e4 e6				
		c4 g6		d4 e6		e4 g6				
		c4 Nc6		d4 g6		e4 Nc6				
		c4 Nf6		d4 Nc6						
				d4 Nf6						

Table 14. Moves 1 and 2 sorted by strength

61- 60	59- 58	57- 56	55- 54	53- 52	51- 50	49- 48	47- 46	45- 44	43- 42	41- 40
e4 e5		e4 Nc6	Nc3 e5	Nf3 d5	c4 e5	e4 d6	d4 c6	c4 c6	d4 g6	d3 g6
e4 e6		e3 Nc6	d4 Nf6	d3 d5	c4 e6	Nc3 d5	d4 c5	e3 b6	d4 b6	d3 b6
		d4 d5		Nf3 Nf6	d4 d6	e3 Nf6	c4 c5	g3 d5	g3 d6	c4 g6
		Nc3 Nc6		e4 d5	d4 Nc6	d3 e5	c4 d6	b3 d5	f4 d6	
				c4 Nc6	d4 e5	Nf3 d6		e3 g6	b3 d6	
				d4 e6	e4 c5	c4 Nf6		e4 g6	b4 d6	
					Nf3 Nc6	c4 d5				
					d3 Nf6					
					e4 c6					
					Nc3 Nf6					

AN EXAMPLE WITH INTREPRETATION

To test how RS predicts moves based on its statistical analysis, an example chess game was chosen randomly from the 12th World Computer Chess Championship in 2004. The Fritz engine competing was not the more powerful Deep Fritz engine for dual processors as used here to prepare a list of the four best moves from which the opposing engine could choose. If the opposing engine made move x_1 and Fritz made move x_2, then Deep Fritz prepares a list of its four best moves y_1, y_2, y_3, and y_4 for the opposing engine. These four moves are then analyzed by RS which returns the three highest statistical values as z_1, z_2, and z_3. Only twenty game moves are considered.

The example is Game 17 for Fritz-Falcon 0-1 in Tab. 15. The Deep Fritz values are the positive or negative fractions preceding the move to show how the best four moves are sorted in descending order.

The bottom row shows which of the strongest three values as specified by RS was selected by Falcon. In Tab. 15, RS picked the move that was played by Falcon in 10 out of 10 moves with the probability of this happening by chance as about (1/3)^4 x (2/3)^5 or 0.016 and 1 in 615.

Table 15. Game 17 with best RS statistics

1	2	3	4	5	6	7	8	9	10
Nf3	g3	Bg2	O-O	d3	c4	cxd5	Nc3	Nd4	e4
d5	c6	Nf6	Bf5	h6	e6	exd5	Be7	Bh7	O-O
0.06 d5	-0.04 Nf6	-0.01 e6	0.05 g6	0.25 Nbd7	0.17 dxc4	0.43 exd5	0.24 Bc5	0.33 Bg6	0.27 O-O
53.51	68.60	59.70	72.17	83.63	97.58	101.32	121.13	99.02	115.35
0.09 Nf6	-0.04 e6		0.08 e6	0.24 e6	0.47 e6	0.48 cxd5	0.38 Be7	0.39 Bg4	
52.59	55.87		70.57	88.16	92.29	92.29	111.33	103.31	
0.24 Nc6		0.02 Nf6			0.49 Qb6				0.36 Nbd7
50.89		74.90			93.80				115.61
	-0.01 c6	0.08 g6	0.15 Bf5	0.26 h6		1.51 Be7	0.60 Be6	0.44 Bh7	0.46 Qb6
	57.00	57.25	92.87	93.75		103.62	107.60	101.11	114.19
1	2	1	1	1	3	2	2	2	2

CONCLUSION

What follows is how to determine when RS should or should not be applied. If chess is ultimately based on the most effective control of space, the reproducible and verifiable basis of RS, then RS theoretically should be applied to all positions. RS may be most effective at the very highest level of abstract play by grand masters as a device to show an advantage in positions considered exactly equal. This means that RS forms a scientific and measurable basis for chess positions that is indifferent by definition to the mechanics of good or bad combinations.

Future work is to be the statistical analysis and classification of recognizable patterns arising from systematic RS evaluation of positions. To that end, over 300 positions are analyzed and classified to a minimum of 8 moves, that is move pairs 1 through 4. Some lines of interest now exist in analyzed form to 12 moves, that is move pairs 1 through 6. Results include definitive move sequence answers to unorthodox openings, such as 1 d4 e5, and new opening move order combinations that do not transpose, are not considered in the tactical chess literature, and are shown to be sound.

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